Oxford Cambridge and RSA

# Tuesday 6 June 2023 – Afternoon

## A Level Mathematics B (MEI)

## H640/01 Pure Mathematics and Mechanics

**Time allowed: 2 hours**

* 9 9 1 7 7 4 5 8 2 8 *

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**QP**

### INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

### ADVICE
- Read each question carefully before you start your answer.

# Formulae A Level Mathematics B (MEI) (H640)

## Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

## Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

## Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

## Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,dx = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

## Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where $\theta$ is measured in radians

**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

**Numerical methods**

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

**Sample variance**

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

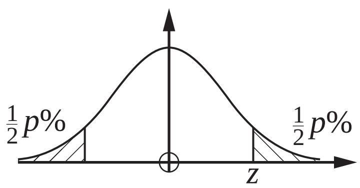
Mean of $X$ is $np$

**Hypothesis testing for the mean of a Normal distribution**

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

**Percentage points of the Normal distribution**

| $p$ | 10 | 5 | 2 | 1 |
|---|---|---|---|---|
| $z$ | 1.645 | 1.960 | 2.326 | 2.576 |




**Kinematics**

| Motion in a straight line | Motion in two dimensions |
|---|---|
| $v = u + at$ | $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ |
| $s = ut + \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ |
| $s = \frac{1}{2}(u + v)t$ | $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ |
| $v^2 = u^2 + 2as$ | |
| $s = vt - \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ |

## **Section A** (23 marks)

**1** A ball is thrown vertically upwards with a speed of $8\,\mathrm{m\,s^{-1}}$.

Find the times at which the ball is 3 m above the point of projection. **[2]**

**2** Express $\dfrac{5x+1}{x^2-x-12}$ in partial fractions. **[4]**

**3** Find $\int\left(2x^4 - x\sqrt{x}\right)\mathrm{d}x$. **[3]**

**4** A ruler PQRS is a uniform rectangular lamina with mass 20 grams. The length of PQ is 30 cm and the length of PS is 4 cm. The ruler is attached at P to a smooth hinge and held with S vertically below P by a horizontal force of magnitude $F$ N as shown in the diagram.


**(a)** Calculate the value of $F$. **[3]**

**(b)** Explain what would happen to the lamina if the force at S were removed. **[1]**

**5** **In this question you must show detailed reasoning.**

**(a)** Find the coordinates of the two stationary points on the graph of $y = 15 - x^2 - \dfrac{16}{x^2}$. **[3]**

**(b)** Show that both these stationary points are maximum points. **[2]**

**6** **(a)** Show that the equation $\sin\left(x+\frac{1}{6}\pi\right)=\cos\left(x-\frac{1}{4}\pi\right)$ can be written in the form

$\tan x=\dfrac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$. **[4]**

**(b)** Hence solve the equation $\sin\left(x+\frac{1}{6}\pi\right)=\cos\left(x-\frac{1}{4}\pi\right)$ for $0 \leqslant x \leqslant 2\pi$. **[1]**

## Section B (77 marks)

7 Determine the exact distance between the two points at which the line through (4, 5) and (6, −1) meets the curve $y = 2x^2 - 7x + 1$. **[7]**

8 A bus is travelling along a straight road at $5.4\,\mathrm{m\,s^{-1}}$. At $t = 0$, as the bus passes a boy standing on the pavement, the boy starts running in the same direction as the bus, accelerating at $1.2\,\mathrm{m\,s^{-2}}$ from rest for $5\,\mathrm{s}$. He then runs at constant speed until he catches up with the bus.

**(a)** The diagram in the Printed Answer Booklet shows the velocity-time graph for the bus.

Draw the velocity-time graph for the boy on this diagram. **[3]**

**(b)** Determine the time at which the boy is running at the same speed as the bus. **[2]**

**(c)** Find the maximum distance between the bus and the boy. **[3]**

**(d)** Find the distance the boy has run when he catches up with the bus. **[3]**

9 The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x - 4\mathrm{e}^{-x}$.

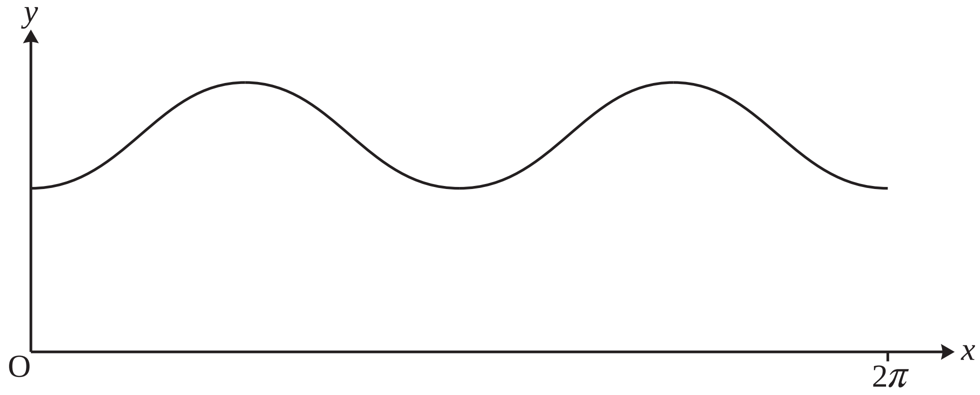
**(a)** Show that the $x$-coordinate of any point on the curve at which the gradient is 3 satisfies the equation $(\mathrm{e}^x)^2 - 3\mathrm{e}^x - 4 = 0$. **[2]**

**(b)** Hence show that there is only one point on the curve at which the gradient is 3, stating the exact value of its $x$-coordinate. **[3]**

**(c)** The curve passes through the point $(0, 0)$.

Show that when $x = 1$ the curve is below the $x$-axis. **[5]**

**10** The diagram shows the graph of $y = 1.5 + \sin^2 x$ for $0 \leqslant x \leqslant 2\pi$.




**(a)** Show that the equation of the graph can be written in the form $y = a - b\cos 2x$ where $a$ and $b$ are constants to be determined. **[2]**

**(b)** Write down the period of the function $1.5 + \sin^2 x$. **[1]**

**(c)** Determine the $x$-coordinates of the points of intersection of the graph of $y = 1.5 + \sin^2 x$ with the graph of $y = 1 + \cos 2x$ in the interval $0 \leqslant x \leqslant 2\pi$. **[3]**

**11** The height $h$ cm of a sunflower plant $t$ days after planting the seed is modelled by $h = a + b\ln t$ for $t \geqslant 9$, where $a$ and $b$ are constants. The sunflower is 10 cm tall 10 days after planting and 200 cm tall 85 days after planting.

**(a)** **(i)** Show that the value of $b$ which best models these values is 88.8 correct to **3** significant figures. **[2]**

**(ii)** Find the corresponding value of $a$. **[1]**

**(b)** **(i)** Explain why the model is not suitable for small positive values of $t$. **[1]**

**(ii)** Explain why the model is not suitable for very large positive values of $t$. **[1]**

**(c)** Show that the model indicates that the sunflower grows to 1 m in height in less than half the time it takes to grow to 2 m. **[2]**

**(d)** Find the value of $t$ for which the rate of growth is 3 cm per day. **[3]**

**12** In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertically upwards respectively.

A particle has mass 2 kg.

**(a)** Write down its weight as a vector. **[1]**

A horizontal force of 3 N in the $\mathbf{i}$ direction and a force $\mathbf{F} = (-4\mathbf{i} + 12\mathbf{j})$ N act on the particle.

**(b)** Determine the acceleration of the particle. **[3]**

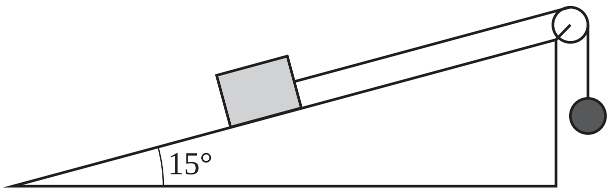
**(c)** The initial velocity of the particle is $5\mathbf{i}\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle after 4 s. **[2]**

**(d)** Find the extra force that must be applied to the particle for it to move at constant velocity. **[1]**

**13** A block of mass 8 kg is placed on a rough plane inclined at 15° to the horizontal. The coefficient of friction between the block and the plane is 0.3.

One end of a light rope is attached to the block. The rope passes over a smooth pulley fixed at the top of the plane, and a sphere of mass 5 kg, attached to the other end of the rope, hangs vertically below the pulley. The part of the rope between the block and the pulley is parallel to the plane. The system is released from rest, and as the sphere falls the block moves directly up the plane with acceleration $a\,\mathrm{m\,s^{-2}}$.




**(a)** On the diagram in the Printed Answer Booklet, show all the forces acting on the block and on the sphere. **[4]**

**(b)** Write down the equation of motion for the sphere. **[2]**

**(c)** Determine the value of $a$. **[6]**

**14** **(a)** Use the laws of logarithms to show that $\log_{10}200 - \log_{10}20$ is equal to 1. **[2]**

The first three terms of a sequence are $\log_{10}20$, $\log_{10}200$, $\log_{10}2000$.

**(b)** Show that the sequence is arithmetic. **[2]**

**(c)** Find the exact value of the sum of the first 50 terms of this sequence. **[2]**

**15** A projectile is launched from a point on level ground with an initial velocity $u$ at an angle $\theta$ above the horizontal.

**(a)** Show that the range of the projectile is given by $\dfrac{2u^2 \sin\theta \cos\theta}{g}$. **[3]**

**(b)** Determine the set of values of $\theta$ for which the maximum height of the projectile is greater than the range, where $\theta$ is an acute angle. Give your answer in degrees. **[5]**

**END OF QUESTION PAPER**

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